Evaluation of Performance Measures of Bulk Arrival Queue With Fuzzy Parameters Using Robust Ranking Technique

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ABSTRACT

This paper proposes a procedure to find the various performance measures in terms of crisp values for bulk arrival queuing systems with varying fuzzy batch sizes where the arrival rate, service rate and batch sizes are fuzzy numbers. Here the inter arrival time, service time and batch sizes are Trapezoidal as well as Triangular fuzzy numbers. The basic idea is to convert the fuzzy inter arrival rate, service rate and batch sizes into crisp values by applying Robust ranking technique. Then apply the crisp values in the classical queuing performance measure formulas. Ranking fuzzy numbers plays a vital role in decision making under fuzzy environment. This ranking technique is a very convenient method, simple to apply and can be used for all types of queuing problems. Illustrations are given to find the performance measures of the characteristics of bulk arrival queuing systems with varying fuzzy batch sizes.

KEY WORDS: *Bulk arrival queues, Fuzzy ranking, fuzzy sets (normal and convex), Membership functions, Trapezoidal fuzzy number, Triangular fuzzy number.*

I. INTRODUCTION

Many of the systems produce arrivals in "bulks", where much number of customers arrive simultaneously at each arrival moment. Investigating bulk arrivals is a direct variation on our basic tagged customer analysis, and serves as a good introduction to make to order modelling.The single server Bulk queues are elaborately studied by many experts like Bailey [1], Bhat[2], Borthakur [3], Chaudhary and Templeton [4]. Bulk arrival queuing models are widely used in several situations such as manufacturing systems, tele communication systems and computer networks [5]. For example in manufacturing, system will not begin till a certain number of raw materials are accumulated during an idle period. We take a frequent analysis of this system by a bulk arrival queuing model which gives a powerful tool for calculating the system performance measures. Within the context of classical queuing theory, the inter arrival time and service times are necessary to follow certain probability distributions. However, in many real life applications, the statistical information may be got subjectively, i.e., the arrival and service mode are more correctly described by linguistic terms such as fast, slow (or) moderate, rather than by probability distributions. Hence fuzzy queues are much more realistic than the regularly used crisp queues. Buckley [6] analysed elementary multiple server queuing models with finite or infinite capacity and calling population. In that the arrivals and departures are followed by possibility distributions. Ranking techniques have been analysed by such researchers like F.Choobinesh and H.Li[7], R.R.Yager[8], S.H.Chen[9]. A. NagoorGani and V. Ashok Kumar[12] have analysed bulk arrival fuzzy queues with fuzzy outputs. In this paper we develop a method that is able to provide performance measures in terms of crisp values for bulk arrival queues with fuzzified exponential arrival rate (i.e. the expected number of arrivals per time period) and service rate (i.e the expected number of services per time period) and varying fuzzy batch sizes. Here Robust ranking technique has been used to attain crisp values.

II. PRELIMINARIES

Fuzzy set was first introduced by Zadeh [10] in 1965.It is a mathematical way of representing Impreciseness or vagueness in everyday real life.

2.1 Definition.

A fuzzy set is characterized by a membership function mapping elements of a domain space, or universe of discourse X to the unit interval [0,1].(i,e) A = {(x, μ_A(x)) ; x ∈ X}, Here μ_A : X →[0,1] is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of x Є X in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0,1].

2.2 Definition.

A fuzzy set A of the universe of discourse X is called a *normal* fuzzy set if there exists atleast one x Є X such that $\mu_A(x) = 1$.

2.3 Definition.

The fuzzy set A is *convex* if and only if for any x_1, x_2 Є X, the membership function of A satisfies the condition $\mu_A\{\lambda x_1+(1-\lambda)x_2\} \geq \min\{\mu_A(x_1), \mu_A(x_2)\}$. $0 \leq \lambda \leq 1$.

2.4 Definition (***Trapezoidal fuzzy number***).

For a Trapezoidal number A(x), it can be represented by A(a,b,c,d;1) with membership function μ(x) given by

$$\mu(x) = \begin{cases} \frac{x-a}{b-a}, a \leq x \leq b \\ 1, b \leq x \leq c \\ \frac{d-x}{d-c}, c \leq x \leq d \\ 0, otherwise \end{cases}$$

2.5 Definition (***Triangular fuzzy number***).

For a triangular number A(x), it can be represented by A(a,b,c;1) with membership function μ(x) given by

$$\mu(x) = \begin{cases} \frac{x-a}{b-a}, a \leq x \leq b \\ 1, x = b \\ \frac{c-x}{c-b}, b \leq x \leq c \\ 0, otherwise \end{cases}$$

2.6 Definition (***α-cut of a fuzzy number***)

The α-cut of a fuzzy number A(x) is defined as

$$A(\alpha) = \{ x : \mu(x) \geq \alpha, \alpha \in [0,1]\}$$

Addition of two Trapezoidal fuzzy numbers can be performed as

$$(a_1, b_1, c_1, d_1)+(a_2, b_2, c_2, d_2) = (a_1+a_2, b_1+b_2, c_1+c_2, d_1+d_2).$$

Addition of two Triangular fuzzy numbers can be performed as

$$(a_1, b_1, c_1)+(a_2, b_2, c_2) = (a_1+a_2, b_1+b_2, c_1+c_2).$$

III. BULK ARRIVAL QUEUES IN FUZZY WITH VARIATION OF FUZZY BATCH SIZES

Consider a bulk arrival queuing system in which customers arrive at a single service facility in batches as a Poisson process with bulk arrival rate $\tilde{\lambda}$ where $\tilde{\lambda}$ is a fuzzy number and all service times follow exponential distribution with fuzzy service rate $\tilde{\mu}$. The bulk arrival with size k is represented by Trapezoidal fuzzy number. Queue discipline is defined as a first-come-first serve (FCFS) and both the size of calling population and the system capacity are infinite. This model will hereafter be denoted by F$M^{[K]}$/FM/1. With the help of the α-cuts, the Trapezoidal arrival size can be represented by different levels of interval of confidences. Take this interval of confidence be $[t_{1\alpha}, t_{2\alpha}]$. Since probability distributions for the α-cut sets can be represented by uniform distributions, we have

$$P(t_\alpha) = \frac{1}{t_{2\alpha}-t_{1\alpha}}, t_{1\alpha} \leq t_\alpha \leq t_{2\alpha}$$

Hence the mean of the distribution is

$$E(T_\alpha) = \int_{t_{1\alpha}}^{t_{2\alpha}} \frac{1}{t_{2\alpha}-t_{1\alpha}} t_\alpha dt_\alpha = \frac{1}{2}(t_{1\alpha} + t_{2\alpha})$$

Similarly for the second moment, we have

$$E(T_\alpha^2) = \int_{t_{1\alpha}}^{t_{2\alpha}} \frac{1}{t_{2\alpha}-t_{1\alpha}} t^2{}_\alpha dt_\alpha = \frac{t_{2\alpha}^3 - t_{1\alpha}^3}{3(t_{2\alpha}-t_{1\alpha})}$$

$$\text{Hence } Var(T_\alpha) = \frac{1}{12} t_{2\alpha} - t_{1\alpha})^2$$

Here the group arrival rate $\tilde{\lambda}$ and service rate $\tilde{\mu}$ are approximately known and can be represented by convex fuzzy sets.

Let $\mu_{\tilde{\lambda}}(x)$ and $\mu_{\tilde{\mu}}(y)$ denote the membership functions of the group arrival rate and service rate respectively.

$$\tilde{\lambda} = \{(x ,\mu_{\tilde{\lambda}}(x))/x \in S(\tilde{\lambda})\}$$

$$\tilde{\mu} = \{(y, \mu_{\tilde{\mu}}(y)/y \in S(\tilde{\mu})\}$$

Where S ($\tilde{\lambda}$) and S ($\tilde{\mu}$) are the supports of $\tilde{\lambda}$ and $\tilde{\mu}$ which denote the universal sets of the arrival rate and service rate respectively. Without loss of generality, We assume that the performance measure is the expected number of customers in the queue Lq. From the known classical queuing theory [4, 5] under the steady-state condition $\rho = xE[K] / y < 1$, where E[K] denotes the expectation of K, the expected number of customers in the queue of a crisp queuing system with bulk arrival is

$$L_q = \frac{x\{yE(K^2) + 2x(E(K)^2) - yE(K)\}}{2y\{y - xE(K)\}}$$

IV. ROBUST RANKING TECHNIQUE – ALGORITHM

To find the Performance measures in terms of crisp values we defuzzify the fuzzy numbers into crisp ones by a fuzzy number ranking method. Robust ranking technique [11] which satisfies compensation, linearity, and additive properties and provides results which are consistent with human intuition. Give a convex fuzzy number ã, the Robust Ranking Index is defined by

$$R(\tilde{a}) = \int_0^1 0.5(a_\alpha{}^L + a_\alpha{}^U)d\alpha$$

Where $(a_\alpha{}^L, a_\alpha{}^U)$ is the α –level cut of the fuzzy number ã.

In this paper we use this method for ranking the fuzzy numbers. The Robust ranking index R(ã) gives the representative value of the fuzzy number ã . It satisfies the linearity and additive property.

V. NUMERICAL EXAMPLE

5.1.Example 1(For Trapezoidal fuzzy number)

Consider a manufacturing system in which jobs arrive in batches. The Trapezoidal arrival size is a trapezoidal fuzzy number $\tilde{K}$= [1, 2, 3, 4] and the interval of confidence be represented by [1+α, 4 - α]. Here the group arrival rate and service rate are Trapezoidal fuzzy numbers represented by $\tilde{\lambda}$=[2, 3, 4, 5] and $\tilde{\mu}$=[13,14,15,16] per minute Whose intervals of confidence are [2+α, 5 - α] and [13+α, 16- α] respectively. The manager of the system wants to calculate the performance measures of the system such as the average number of jobs in queue.

Now we evaluate R(1,2,3,4) by applying Robust ranking method. The membership function of the Trapezoidal fuzzy number (1, 2, 3, 4) is

$$\mu(x) = \begin{cases} \frac{x-1}{1}, 1 \le x \le 2 \\ 1, 2 \le x \le 3 \\ \frac{4-x}{1}, 3 \le x \le 4 \\ 0, otherwise \end{cases}$$

The α –cut of the fuzzy number (1, 2, 3, 4) is $(a_\alpha{}^L, a_\alpha{}^U)$ = (α+1, 4-α) for which

$$R(\tilde{K}) = R(1,2,3,4) = \int_0^1 0.5(a_\alpha{}^L, + a_\alpha{}^U)d\alpha = \int_0^1 0.5(5)d\alpha = 2.5$$

Proceeding similarly, the Robust Ranking Indices for the fuzzy numbers $\tilde{\lambda}$, $\tilde{\mu}$ are calculated as:

$$R(\tilde{\lambda}) = 3.5, R(\tilde{\mu}) = 14.5$$

$$E(K)=E(2.5)=2.5, E(K^2)=E(6.25)=6.25$$

It is clear that in this example the steady-state condition ρ=xE(K)/y<1 is satisfied

$$\text{From } L_q=\frac{x\{yE(K^2)+2x(E(K)^2)-yE(K)\}}{2y\{y-xE(K)\}}$$

$$L_q=\frac{3.5\{14.5(6.25)+2(3.5)(2.5^2)-14.5(2.5)\}}{2(14.5)\{14.5-3.5(2.5)\}}=2.059$$

Using Little's Formula

$$L_s= L_q+\frac{\lambda}{\mu}=2.059+\frac{3.5}{14.5}=2.059+0.2413=2.3003$$

$$W_q=\frac{L_q}{\lambda}=\frac{2.059}{3.5}=0.588 \text{ minutes}$$

$$W_s=\frac{L_s}{\lambda}=\frac{2.3003}{3.5}=0.6572 \text{ minutes}$$

5.2. Example 2(For Triangular fuzzy number)

Consider a manufacturing system in which jobs arrive in batches. Using α-cuts, the Triangular arrival size is a Triangular fuzzy number $\tilde{K}$= [1, 2, 4] and the interval of confidence be represented by [1+α, 4 - 2α]. Both the group arrival rate and service rate are Triangular fuzzy numbers represented by $\tilde{\lambda}$=[2, 3, 5]and $\tilde{\mu}$=[13,14,16] per minute Whose intervals of confidence are [2+α, 5 - 2α] and [13+α, 16-2α] respectively. The system manager wants to evaluate the performance measures of the system such as the expected number of jobs in queue. Now we calculate R(1,2,4) by applying Robust ranking method. The membership function of the Triangular fuzzy number (1, 2,4) is

$$\mu(x)=\begin{cases}\frac{x-1}{1}, 1\le x\le 2\\ 1, \quad x=2\\ \frac{4-x}{2}, 2\le x\le 4\\ 0, otherwise\end{cases}$$

The α –cut of the fuzzy number (1, 2, 4) is $(a_\alpha{}^L, a_\alpha{}^U)$ = (α+1,4-2α) for which

$$R(\tilde{K})= R(1,2,4) =\int_0^1 0.5(a_\alpha{}^L,+a_\alpha{}^U)d\alpha=\int_0^1 0.5(5-\alpha)d\alpha = 0.5[5\alpha-\frac{\alpha^2}{2}]_0^1=0.5[\frac{9}{2}]=2.25$$

Proceeding similarly, the Robust Ranking Indices for the fuzzy numbers $\tilde{\lambda}$, $\tilde{\mu}$ are calculated as:

$$R(\tilde{\lambda})= 3.25, R(\tilde{\mu})=14.25$$

$$E(K)=E(2.25)=2.25, E(K^2)=E(5.062)=5.062$$

It is clear that in this example the steady-state condition ρ=xE(K)/y<1 is satisfied

$$From \quad L_q=\frac{x\{yE(K^2)+2x(E(K)^2)-yE(K)\}}{2y\{y-xE(K)\}}$$

$$L_q=\frac{3.25\{14.25(5.062)+2(3.25)(2.25^2)-14.25(2.25)\}}{2(14.25)\{14.25-3.25(2.25)\}}=1.1996$$

Using Little's Formula

$$L_s= L_q+\frac{\lambda}{\mu}=2.059+\frac{3.5}{14.5}=1.1996+0.2280=1.4276$$

$$W_q=\frac{L_q}{\lambda}=\frac{1.1996}{3.25}=0.3691 \text{ minutes}$$

$$W_s=\frac{L_s}{\lambda}=\frac{1.4276}{3.25}=0.4392 \text{ minutes}$$

VI. CONCLUSION

In this paper, Fuzzy set theory has been applied to bulk arrival queues. Bulk arrival queuing models have been used in operations and service mechanism for evaluating system performance. This paper develops a method to find the crisp values of performance measures of bulk arrival queues where the batch arrival size, arrival rate and service rate are fuzzy which are more realistic and general in nature. Moreover, the fuzzy problem has been transformed into crisp problem using Robust ranking technique.Since the performance measures such as the system length, the waiting time are crisp values, the manager can take the best and optimum decisions. One can conclude that the solution of fuzzy problems can be obtained by Robust ranking method effectively. The technique proposed in this paper provides realistic information for system manager.

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